

New Approach to Hard Corrections in Precision QCD for LHC and FCC Physics

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Abstract

We present a new approach to the realization of hard fixed-order corrections in predictions for the processes probed in high energy colliding hadron beam devices, with some emphasis on the LHC and the future FCC devices. We show that the usual unphysical divergence of such corrections as one approaches the soft limit is removed in our approach, so that we would render the standard results to be closer to the observed exclusive distributions. We use the single Z/γ^* production and decay to lepton pairs as our prototypical example, but we stress that the approach has general applicability. In this way, we open another part of the way to rigorous baselines for the determination of the theoretical precision tags for LHC physics, with an obvious generalization to the future FCC as well.

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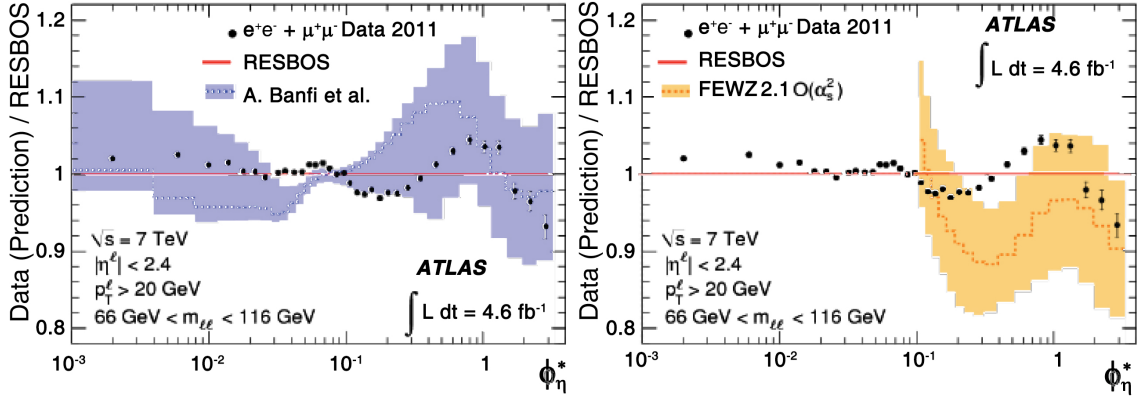
Now that we have entered the era of precision QCD, by which we mean predictions for QCD processes at the total precision tag of 1% or better, it is paramount to have rigorous baselines with respect to which to compare theoretical results both against one another and against the new LHC precision data as well as for expectations for the future FCC [1] device. For example, we have argued in Refs. [2–5] that exact, amplitude-based resummation allows one to have better than 1% theoretical precision as a realistic goal in such comparisons as those needed in determining the detailed properties of the newly discovered BEH [6] boson [7], so that one can indeed distinguish new physics(NP) from higher order SM processes and can distinguish different models of new physics from one another as well. One of the ingredients in exact amplitude-based resummation is the respective set of hard gluon residuals which determine the order of exactness in the respective QCD predictions. These residuals obtain from exact fixed-order QCD perturbation theory and thus any attempt to determine their precision tag necessarily entails determining the respective precision tag of the corresponding fixed-order results. Unfortunately, in the current state of the art, even though we have for a process such as single Z/γ^* production at LHC(FCC) even the NNLO exact result [8], when one tries to compare the predicted p_T (or ϕ_η^*)¹ spectrum with the LHC and FNAL data, one sees the type of divergence shown in Figs. 1 and 2 taken from Refs. [10,11]². The “soft” limit of the prediction has no reasonable relation to that in the data and even at $p_T \cong 20\text{GeV}$ the so-called exact NNLO result is just useless. Obviously, it will make no sense to talk about the precision tag of such results! Indeed, one of the consequences of the discrepancy in Figs. 1, 2 is that, until one understands how to fix it, one cannot even be sure of the normalization one gets when one integrates over the theoretical prediction itself. Indeed, in the precision theory developed and implemented [17] for the LEP physics program, it was in fact true that “fixing” such discrepancies changed the normalization.

More precisely, in Refs. [3,4], we have shown that the current realization of the exact amplitude-based resummation approach to precision LHC physics as effected by the implementation of the IR-improved DGLAP-CS [18,19] theory [20,21] via HERWIRI1.031 [4] in the HERWIG6.5 [22] environment improves the agreement between LHC data on single Z/γ^* production in comparison to the un-improved predictions. This prepares the stage naturally for setting baselines for the respective theoretical precision tags especially when we focus on the NLO exact, matrix element matched parton shower MC precision issues involved in comparing the predictions of MC@NLO/HERWIRI1.031 and MC@NLO/HERWIG6.5 in the MC@NLO [23] methodology. Here, we define MC@NLO/A to be the NLO exact, matrix element matched parton shower MC realization of MC A

¹ Here, ϕ_η^* is a new p_T -related variable [9] used in some of the comparisons with data and it is defined as follows: $\phi_\eta^* = \tan(\frac{1}{2}(\pi - \Delta\phi)) \sin \theta^* \cong \left| \sum \frac{p_{iT} \sin \phi_i}{Q} \right| + \mathcal{O}(\frac{p_{iT}^2}{Q^2})$, where $\Delta\phi = \phi_1 - \phi_2$ is the azimuthal angle between the two leptons which have transverse momenta \vec{p}_{iT} , $i = 1, 2$, and θ^* is the scattering angle of the dilepton system relative to the beam direction when one boosts to the frame along the beam direction such that the leptons are back to back.

²Note that in Fig. 2 the comparisons with RESBOS [12–14], which realizes the “CSS” resummation in Ref. [15], show that in the regime where fixed-order result takes over from the resummed terms we see the prediction overshoot the data – see also the discussion below from Ref. [16].

Z/γ^* transverse momentum ($d\sigma/d\phi_\eta^*(\ell\ell)$)



- Calculations from A. Banfi et al. (resummed QCD predictions+fixed-order pQCD) is less good than Resbos
- **Measurement precision about one order of magnitude lower than the present theoretical uncertainties**
- FEWZ predictions undershoot the data by ~10% which confirm previous CDF observation (PRD 86,052010)

Figure 1: Comparisons of some theoretical predictions with the ATLAS $Z/\gamma^* \phi_\eta^*$ spectrum in single Z/γ^* production with decay to lepton pairs as given in Ref. [10]. Here Banfi *et al.* refers also to a resummed calculation of the “CSS” type [15], so that it has the same physical precision limitations as RESBOS [12–14] as discussed in Ref. [3] – see the second reference in Refs. [9].

in the MC@NLO methodology. When we try to address these issues, we are faced with determining the precision of the respective NLO exact matrix element prediction. This latter issue then brings us to the NLO version of the type of behavior discussed above for Figs. 1, 2, which is illustrated in Fig. 3 and discussed at length in Ref. [16] where in its eq.(5.5.30) it is shown that

$$\frac{G_{p \rightarrow q}^{DY}(x, Q^2)}{G_{p \rightarrow q}(x, Q^2)} \xrightarrow{x \rightarrow 1} 1 + \frac{2\alpha_s(Q^2)}{3\pi} \ln^2(1-x) \quad (1)$$

where $G_{p \rightarrow q}^{DY}$ ($G_{p \rightarrow q}$) is the respective Drell-Yan(DIS) structure function [16] in a standard type of notation. No observable data, at LHC or the new FCC, can have this behavior and it calls into question what a precision tag could even mean here?

With an eye toward “taming” this what we see in Fig. 3 and (1), we revisit our master formula from Refs. [5] for our $QED \otimes QCD$ exact resummation theory

$$\begin{aligned} d\sigma_{\text{res}} = & e^{\text{SUM}_{\text{IR}}(\text{QCED})} \sum_{n,m=0}^{\infty} \frac{1}{n!m!} \int \prod_{j_1=1}^n \frac{d^3 k_{j_1}}{k_{j_1}} \\ & \prod_{j_2=1}^m \frac{d^3 k'_{j_2}}{k'_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy \cdot (p_1 + q_1 - p_2 - q_2 - \sum k_{j_1} - \sum k'_{j_2}) + D_{\text{QCED}}} \end{aligned}$$

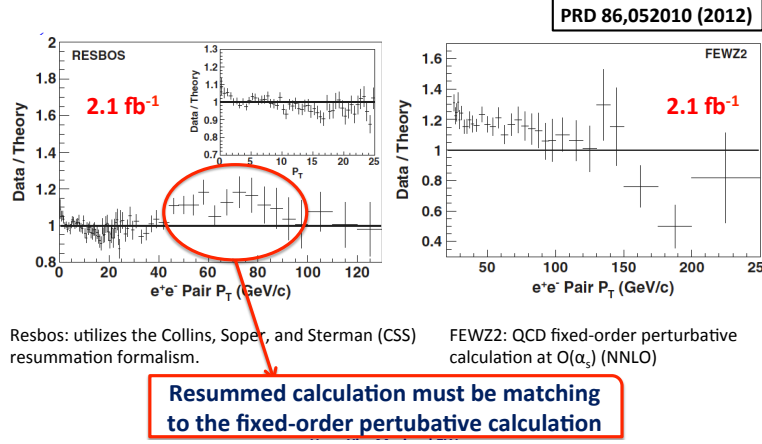


Figure 2: Comparisons of theoretical predictions with the CDF Z/γ^* p_T spectrum in single Z/γ^* production with decay to lepton pairs as given in Ref. [11].

$$\tilde{\beta}_{n,m}(k_1, \dots, k_n; k'_1, \dots, k'_m) \frac{d^3 p_2}{p_2^0} \frac{d^3 q_2}{q_2^0}, \quad (2)$$

where $d\bar{\sigma}_{\text{res}}$ is either the reduced cross section $d\hat{\sigma}_{\text{res}}$ or the differential rate associated to a DGLAP-CS [18, 19] kernel involved in the evolution of PDF's and where the *new* (YFS-style [17, 24]) *non-Abelian* residuals $\tilde{\beta}_{n,m}(k_1, \dots, k_n; k'_1, \dots, k'_m)$ have n hard gluons and m hard photons and we show the final state with two hard final partons with momenta p_2, q_2 specified for a generic $2f$ final state for definiteness. The infrared functions $\text{SUM}_{\text{IR}}(\text{QCD})$, D_{QCD} are defined in Refs. [5, 20, 21] as follows:

$$\begin{aligned} \text{SUM}_{\text{IR}}(\text{QCD}) &= 2\alpha_s \Re B_{\text{QCD}}^{nls} + 2\alpha_s \tilde{B}_{\text{QCD}}^{nls} \\ D_{\text{QCD}} &= \int \frac{d^3 k}{k^0} (e^{-iky} - \theta(K_{\text{max}} - k^0)) \tilde{S}_{\text{QCD}}^{nls} \end{aligned} \quad (3)$$

where the dummy parameter K_{max} is such that nothing depends on it. We have introduced

$$\begin{aligned} B_{\text{QCD}}^{nls} &\equiv B_{\text{QCD}}^{nls} + \frac{\alpha}{\alpha_s} B_{\text{QED}}^{nls}, \\ \tilde{B}_{\text{QCD}}^{nls} &\equiv \tilde{B}_{\text{QCD}}^{nls} + \frac{\alpha}{\alpha_s} \tilde{B}_{\text{QED}}^{nls}, \\ \tilde{S}_{\text{QCD}}^{nls} &\equiv \tilde{S}_{\text{QCD}}^{nls} + \tilde{S}_{\text{QED}}^{nls}. \end{aligned} \quad (4)$$

The DGLAP-CS synthesization of the infrared functions is denoted by the superscript *nls* as explained in Refs. [5, 20, 21, 25] while the infrared functions $B_A, \tilde{B}_A, \tilde{S}_A$, $A = \text{QCD}, \text{QED}$, are given in Refs. [5, 17, 20, 21, 24]. The exactness of the simultaneous resummation of QED and QCD large IR effects that we show here cannot be emphasized too much.

In the interest of pedagogy, we note that, in the language of Ref. [26], the exponent $\text{SUM}_{\text{IR}}(\text{QCD})$ sums up to the infinite order the maximal leading IR singular terms in the

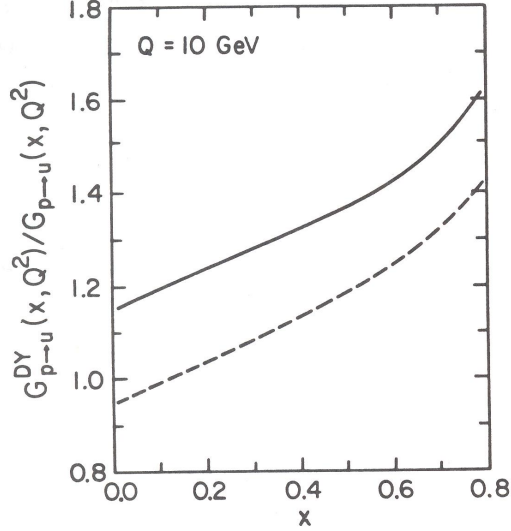


Figure 3: The ratio of the u-quark probability distribution defined in the Drell-Yan process to that defined from the F_2 structure function defined in deep inelastic lepton-nucleon scattering as discussed in Ref. [16] at $Q = 10$ GeV, where the solid (dashed) curve corresponds to including (excluding) the total cross section in the attendant Drell-Yan distribution.

cross section for soft emission below a dummy parameter K_{\max} and the exponent D_{QCED} does the same for the regime above K_{\max} so that (2) is independent of K_{\max} ³. Exactness order by order in perturbation theory in both α and α_s in the presence of these resummed terms, as explained in Refs. [5, 20, 21], is maintained by iterative computation of the residuals $\tilde{\beta}_{n,m}$ to match the attendant exact results to all orders in α and α_s . In particular, in our formulation in (2) *the entire soft gluon phase space is included in the representation – no part of it is dropped*. As it is shown in Refs. [5], the new non-Abelian residuals $\tilde{\beta}_{m,n}$ carry a realization of rigorous shower/ME matching via their shower subtracted analogs: in (2) we make the replacements

$$\tilde{\beta}_{n,m} \rightarrow \hat{\tilde{\beta}}_{n,m} \quad (5)$$

where the $\hat{\tilde{\beta}}_{n,m}$ have had all effects in the showers associated to the attendant PDF's $\{F_j\}$ removed from them. Here we have in mind the standard formula for the fully differential representation of a hard LHC(FCC) scattering process:

$$d\sigma = \sum_{i,j} \int dx_1 dx_2 F_i(x_1) F_j(x_2) d\hat{\sigma}_{\text{res}}(x_1 x_2 s), \quad (6)$$

³If we want to include more of the maximal exponentiating terms from the formalism of Ref. [26] in the two exponents $\text{SUM}_{\text{IR}}(\text{QCED})$, D_{QCED} , we may do so with a consequent change in the attendant residuals $\tilde{\beta}_{n,m}$.

where $d\hat{\sigma}_{\text{res}}$ is given in (2) and thus is consistent [2–5] with our achieving a total precision tag of 1% or better for the total theoretical precision of (6).

For completeness, we also recall the connection between our constructs in the master formula (2) and the constructs in the MC@NLO methodology: We may represent the MC@NLO differential cross section via [23]

$$d\sigma_{\text{MC@NLO}} = \left[B + V + \int (R_{MC} - C) d\Phi_R \right] d\Phi_B [\Delta_{MC}(0) + \int (R_{MC}/B) \Delta_{MC}(k_T) d\Phi_R] + (R - R_{MC}) \Delta_{MC}(k_T) d\Phi_B d\Phi_R \quad (7)$$

where B is Born distribution, V is the regularized virtual contribution, C is the corresponding counter-term required at exact NLO, R is the respective exact real emission distribution for exact NLO, $R_{MC} = R_{MC}(P_{AB})$ is the parton shower real emission distribution so that the Sudakov form factor is

$$\Delta_{MC}(p_T) = e^{[- \int d\Phi_R \frac{R_{MC}(\Phi_B, \Phi_R)}{B} \theta(k_T(\Phi_B, \Phi_R) - p_T)]},$$

where as usual it describes the respective no-emission probability. The respective Born and real emission differential phase spaces are denoted by $d\Phi_A$, $A = B, R$. We find it very important *still* to emphasize that the representation of the differential distribution for MC@NLO in (7) illustrates the compensation between real and virtual divergent soft effects discussed in the Appendices of Refs. [20, 21] in establishing the validity of (2) for QCD. More specifically, from comparison with (2) restricted to its QCD aspect we get the identifications, accurate to $\mathcal{O}(\alpha_s)$,

$$\begin{aligned} \frac{1}{2} \hat{\hat{\beta}}_{0,0} &= \bar{B} + (\bar{B}/\Delta_{MC}(0)) \int (R_{MC}/B) \Delta_{MC}(k_T) d\Phi_R \\ \frac{1}{2} \hat{\hat{\beta}}_{1,0} &= R - R_{MC} - B \tilde{S}_{QCD} \end{aligned} \quad (8)$$

where we defined [23]

$$\bar{B} = B(1 - 2\alpha_s \Re B_{QCD}) + V + \int (R_{MC} - C) d\Phi_R$$

and we understand that the DGLAP-CS kernels in R_{MC} are to be taken as the IR-improved ones as we derived in Refs. [20, 21]. Although we have suppressed the superscript *nls* for simplicity of notation, to avoid double counting of effects the QCD virtual and real infrared functions B_{QCD} and \tilde{S}_{QCD} are understood to be DGLAP-CS synthesized as explained in Refs. [5, 20, 21]. Most importantly, in view of (8), we observe that the way to the extension of frameworks such as MC@NLO to exact higher orders in $\{\alpha_s, \alpha\}$ is open via our $\hat{\hat{\beta}}_{n,m}$ and will be taken up elsewhere [27].

We see from the relationship between the hard gluon residuals and the exact NLO corrections that a serious study of the theoretical precision of (6) when it uses the results

of (2), such as it is done in Refs. [4], necessarily involves the studying of the theoretical precision of these exact NLO results and if we have the behavior in (1) we do have to ask what would such a study mean in relation to LHC (or FCC) data? To address this question, we proceed as follows.

We recall the well-known representation of the exact NLO differential cross section for the Drell-Yan process (we focus on the γ^* part of the Z/γ^* exchange for simplicity of presentation, as adding in the effect of the Z is straightforward and does not affect the analysis here in any essential way; similarly, we treat the simple case of one flavor with unit charge following Ref. [28, 29] for the same reason – inserting the proper charges and sums is trivial)

$$\begin{aligned} \frac{d\sigma^{DY}}{dQ^2} = & \frac{4\pi\alpha^2}{9sQ^2} \int_0^1 \frac{dx_1}{x_1} \int_0^1 \frac{dx_2}{x_2} \{ [q^{(1)}(x_1)\bar{q}^{(2)}(x_2) + (1 \leftrightarrow 2)] [\delta(1 - z_{12}) \\ & + \alpha_s(t)\theta(1 - z_{12})\left(\frac{1}{2\pi}P_{qq}(z_{12})(2t) + f_q^{DY}(z_{12})\right)] \\ & + [(q^{(1)}(x_1) + \bar{q}^{(1)}(x_1))G^{(2)}(x_2) + (1 \leftrightarrow 2)] \\ & \times [\alpha_s(t)\theta(1 - z_{12})\left(\frac{1}{2\pi}P_{qG}(z_{12})t + f_G^{DY}(z_{12})\right)] \} \end{aligned} \quad (9)$$

where $z_{12} = \tau/(x_1x_2)$, $\tau = Q^2/s$ in the usual conventions [16, 28, 29], the labels 1 and 2 refer to the two respective incoming protons and we follow the generic notation of Refs. [16, 28] here. The unimproved DGLAP-CS [18, 19] kernels in (9) are well-known as

$$\begin{aligned} P_{qq}(z) &= C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z) \right], \\ P_{qG}(z) &= \frac{1}{2}(z^2 + (1-z)^2), \end{aligned} \quad (10)$$

where we define $t = \ln(Q^2/\mu^2)$ following Refs. [16, 28] so that μ is the 't Hooft [30] unity of mass. The scheme dependent hard correction terms are given as follows [28, 29] if one uses massless quarks and gluons and dimensional regularization, for example:

$$\begin{aligned} \alpha_s f_G^{DY}(z) &= \frac{\alpha_s}{2\pi} \frac{1}{2} [(z^2 + (1-z)^2) \ln \frac{(1-z)^2}{z} - \frac{3}{2}z^2 + z + \frac{3}{2} + 2P_{qG}(z)\zeta] \\ \alpha_s f_q^{DY}(z) &= C_F \frac{\alpha_s}{2\pi} \left[4(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ - 2 \frac{1+z^2}{1-z} \ln z \right. \\ &\quad \left. + \left(\frac{2\pi^2}{3} - 8 \right) \delta(1-z) + \frac{2}{C_F} P_{qq}(z)\zeta \right] \end{aligned} \quad (11)$$

where we define [29] $\zeta = -\frac{1}{\epsilon} + C_E - \ln 4\pi$ for $\epsilon = 2 - n/2$ when n is the dimension of space-time. C_E is Euler-Mascheroni constant. In the $\overline{\text{MS}}$ scheme, the terms proportional to ζ are removed by mass factorization, which also replaces μ by Λ in t following Ref. [16]. This leaves the $+$ -functions in the hard corrections and it is the divergent behavior of these distributions as $z \rightarrow 1$ that produces the attendant unphysical results referenced above. How can we fix this?

We imbed the calculation of the hard correction terms into the master formula (2) restricted to its QCD aspect. This gives the following resummed version of (9):

$$\begin{aligned}
\frac{d\sigma_{res}^{DY}}{dQ^2} = & \frac{4\pi\alpha^2}{9sQ^2} \int_0^1 \frac{dx_1}{x_1} \int_0^1 \frac{dx_2}{x_2} \{ [q^{(1)}(x_1)\bar{q}^{(2)}(x_2) + (1 \leftrightarrow 2)] 2\gamma_q F_{YFS}(2\gamma_q)(1-z_{12})^{2\gamma_q-1} e^{\delta_q} \\
& \times \theta(1-z_{12}) [1 + \gamma_q - 7C_F \frac{\alpha_s}{2\pi} + (1-z_{12})(-1 + \frac{1-z_{12}}{2}) \\
& + 2\gamma_q(-\frac{1-z_{12}}{2} - \frac{z_{12}^2}{4} \ln z_{12}) \\
& + \alpha_s(t) \frac{(1-z_{12})}{2\gamma_q} f_q^{DY}(z_{12})] \\
& + [(q^{(1)}(x_1) + \bar{q}^{(1)}(x_1))G^{(2)}(x_2) + (1 \leftrightarrow 2)] \\
& \times \gamma_G F_{YFS}(\gamma_G) e^{\frac{\delta_G}{2}} [\alpha_s(t)\theta(1-z_{12}) (\frac{t}{2\pi\gamma_G} (\frac{1}{2}(z_{12}^2(1-z_{12})^{\gamma_G} + (1-z_{12})^2 z_{12}^{\gamma_G})) \\
& + f_G^{DY'}(z_{12})/\gamma_G)] \}
\end{aligned} \tag{12}$$

where we have introduced here

$$\begin{aligned}
\alpha_s f_G^{DY'}(z) = & \frac{\alpha_s}{2\pi} \frac{1}{2} [(z^2(1-z)^{\gamma_G} + (1-z)^2 z^{\gamma_G}) \ln \frac{(1-z)^2}{z} - \frac{3}{2} z^2(1-z)^{\gamma_G} + z(1-z)^{\gamma_G} \\
& + \frac{3}{4} ((1-z)^{\gamma_G} + z^{\gamma_G})],
\end{aligned} \tag{13}$$

and the following exponents and YFS infrared function, F_{YFS} , already needed for the IR-improvement of DGLAP-CS theory in Refs. [20, 21]:

$$\begin{aligned}
\gamma_q &= C_F \frac{\alpha_s}{\pi} t = \frac{4C_F}{\beta_0}, & \delta_q &= \frac{\gamma_q}{2} + \frac{\alpha_s C_F}{\pi} (\frac{\pi^2}{3} - \frac{1}{2}), \\
\gamma_G &= C_G \frac{\alpha_s}{\pi} t = \frac{4C_G}{\beta_0}, & \delta_G &= \frac{\gamma_G}{2} + \frac{\alpha_s C_G}{\pi} (\frac{\pi^2}{3} - \frac{1}{2}), \\
F_{YFS}(\gamma) &= \frac{e^{-C_E \gamma}}{\Gamma(1+\gamma)}.
\end{aligned} \tag{14}$$

We define $\beta_0 = 11 - \frac{2}{3}n_f$ for n_f active flavors in a standard way and $\Gamma(w)$ is Euler's gamma function of the complex variable w . Note that we have mass factorized in (12) and (13) as indicated above. It can be seen immediately that the regime at $z_{12} \rightarrow 1$ is now under control in (12) so that we will no longer have the unphysical behavior discussed above. This is the main result of this paper.

Specifically, instead of the result in (1), we now get the behavior such that the $\ln^2(1-x)$ on the RHS of (1) is replaced by

$$\frac{2(1-x)^{\gamma_q} \ln(1-x)}{\gamma_q} - \frac{2(1-x)^{\gamma_q}}{\gamma_q^2},$$

and this vanishes for $x \rightarrow 1$. What our result means that the hard correction now has the possibility to be compared *exclusively* to the data in a rigorously meaningful way. We take up such matters elsewhere. [27].

We stress that the parton shower/ME matching formulas in MC@NLO (shown above) and in POWHEG [31] do not remove the IR divergence which we just tamed, as the latter retains the NLO correction with its bad IR limit in the soft regime for $z_{12} \rightarrow 1$ and the former replaces the bad IR behavior of the NLO correction in the soft $z_{12} \rightarrow 1$ limit with that of the parton shower real emission at the same order and it is well known that the respective unimproved parton shower real emission is infrared divergent for $z_{12} \rightarrow 1$ and requires an ad hoc IR cut-off k_0 -parameter, as we have discussed in Ref. [4]. *No such parameter is needed in our new approach.*

To sum up, we have introduced a new approach to hard corrections in perturbative QCD that will allow us to establish the same type of semi-analytical baselines for QCD that we had in Refs. [17] for the higher order corrections in the Standard Model EW theory. We look forward to its exploitation in precision LHC and FCC physics scenarios. In closing, we thank Prof. Ignatios Antoniadis for the support and kind hospitality of the CERN TH Unit while part of this work was completed.

References

- [1] R. Heuer, FCC-GOV-PM-001, CERN, 2013; see also G. Ambrosio *et al.*, FNAL-TM-2149 (2001); W. Scandale and F. Zimmermann, Nucl. Phys. B Proc. Suppl. **177-178** (2008) 207; P. Limon, in eConf/C010107; G. Dugan and M. Syphers, CBN-99-15 (1999); A.D. Kovalenko, in *Tsukuba 2001, High Energy Accelerators*, p2hc05; P. McIntyre and A. Sattarov, in *Proc. Beyond 2010*, eds. H.V. Klapdor-Kleingrothaus *et al.*, (World Scientific Publ. Co., Singapore, 2011) p. 100; S. Assadi *et al.*, arXiv:1402.5973; and references therein.
- [2] B.F.L. Ward *et al.*, PoS(**RADCOR2013**)(2014) 054.
- [3] S.K. Majhi *et al.*, arXiv:1305.0023.
- [4] S. Joseph *et al.*, Phys. Lett. B**685** (2010) 283; Phys. Rev. D**81** (2010) 076008; B.F.L. Ward *et al.*, Mod. Phys. Lett. A**25** (2010) 2207; B.F.L. Ward and S. Yost, PoS (**ICHEP2010**) (2011) 127; B.F.L. Ward, S.K. Majhi and S.A. Yost, PoS(**RADCOR2011**) (2012) 022; S.K. Majhi *et al.*, Phys. Lett. B**719** (2013) 367; and references therein.
- [5] C. Glosser, S. Jadach, B.F.L. Ward and S.A. Yost, Mod. Phys. Lett. A **19**(2004) 2113; B.F.L. Ward, C. Glosser, S. Jadach and S.A. Yost, in *Proc. DPF 2004*, Int. J. Mod. Phys. A **20** (2005) 3735; in *Proc. ICHEP04, vol. 1*, eds. H. Chen *et al.*, (World. Sci. Publ. Co., Singapore, 2005) p. 588; B.F.L. Ward and S. Yost, preprint BU-HEPP-05-05, in *Proc. HERA-LHC Workshop*, CERN-2005-014; in *Moscow 2006, ICHEP, vol. 1*, p. 505; Acta Phys. Polon. B **38** (2007) 2395; arXiv:0802.0724,

- PoS(**RADCOR2007**)(2007) 038; B.F.L. Ward *et al.*, arXiv:0810.0723, in *Proc. ICHEP08*; arXiv:0808.3133, in *Proc. 2008 HERA-LHC Workshop*, DESY-PROC-2009-02, eds. H. Jung and A. De Roeck, (DESY, Hamburg, 2009)pp. 180-186, and references therein.
- [6] F. Englert and R. Brout, *Phys. Rev. Lett.* **13** (1964) 312; P.W. Higgs, *Phys. Lett.* **12** (1964) 132; *Phys. Rev. Lett.* **13** (1964) 508; G.S. Guralnik, C.R. Hagen and T.W.B. Kibble, *ibid.* **13** (1964) 585.
 - [7] F. Gianotti, in *Proc. ICHEP2012*, in press; J. Incandela, *ibid.*, 2012, in press; G. Aad *et al.*, *Phys. Lett. B* **716** (2012) 1, arXiv:1207.7214; D. Abbaneo *et al.*, *ibid.* **716** (2012) 30, arXiv:1207.7235.
 - [8] K. Melnikov and F. Petriello, *Phys. Rev. D* **74**(2006)114017; R. Gavin *et al.*, arXiv:1101.3540; and references therein.
 - [9] A. Banfi *et al.*, *Eur. Phys. J. C* **71** (2011) 1600; *Phys. Lett. B* **715**(2012) 152 and references therein.
 - [10] S. Hassani, in *Proc. Recntres de Moriond EW, 2013*, in press; G. Aad *et al.*, arXiv:1211.6899, and references therein.
 - [11] H. Yin, in *Proc. Recntres de Moriond EW, 2013*, in press; T. Aaltonen *et al.*, *Phys. Rev. D* **86** (2012) 052010, and references therein.
 - [12] C. Balazs and C.P. Yuan, *Phys. Rev. D* **56** (1997) 5558.
 - [13] G.A. Ladinsky and C.P. Yuan, *Phys. Rev. D* **50** (1994) 4239.
 - [14] F. Landry *et al.*, *Phys. Rev. D* **67** (2003) 073016.
 - [15] J.C. Collins, D.E. Soper and G. Sterman, *Nucl. Phys. B* **250**(1985) 199; in *Les Arcs 1985, Proceedings, QCD and Beyond*, pp. 133-136.
 - [16] R.D. Field, *Applications of Perturbative QCD*, (Addison-Wesley Publ. Co., Inc, Redwood City, 1989).
 - [17] See also S. Jadach and B.F.L. Ward, *Comput. Phys. Commun.* **56**(1990) 351; *Phys. Lett. B* **274** (1992) 470; S. Jadach *et al.*, *Comput. Phys. Commun.* **102** (1997) 229; S. Jadach, W. Placzek and B.F.L. Ward, *Phys. Lett. B* **390** (1997) 298; S. Jadach, M. Skrzypek and B.F.L. Ward, *Phys. Rev. D* **55** (1997) 1206; S. Jadach, W. Placzek and B.F.L. Ward, *Phys. Rev. D* **56** (1997) 6939; S. Jadach, B.F.L. Ward and Z. Was, *Phys. Rev. D* **63** (2001) 113009; *Comp. Phys. Commun.* **130** (2000) 260; *ibid.* **124** (2000) 233; *ibid.* **79** (1994) 503; *ibid.* **66** (1991) 276; S. Jadach *et al.*, *ibid.* **140** (2001) 432, 475; see also S. Jadach, B.F.L. Ward and Z. Was, *Phys. Rev. D* **88** (2013) 114022.

- [18] G. Altarelli and G. Parisi, *Nucl. Phys.* **B126** (1977) 298; Yu. L. Dokshitzer, *Sov. Phys. JETP* **46** (1977) 641; L. N. Lipatov, *Yad. Fiz.* **20** (1974) 181; V. Gribov and L. Lipatov, *Sov. J. Nucl. Phys.* **15** (1972) 675, 938; see also J.C. Collins and J. Qiu, *Phys. Rev. D* **39** (1989) 1398.
- [19] C.G. Callan, Jr., *Phys. Rev. D* **2** (1970) 1541; K. Symanzik, *Commun. Math. Phys.* **18** (1970) 227, and in *Springer Tracts in Modern Physics*, **57**, ed. G. Hoehler (Springer, Berlin, 1971) p. 222; see also S. Weinberg, *Phys. Rev. D* **8** (1973) 3497.
- [20] B.F.L. Ward, *Adv. High Energy Phys.* **2008** (2008) 682312.
- [21] B.F.L. Ward, *Ann. Phys.* **323** (2008) 2147.
- [22] G. Corcella *et al.*, hep-ph/0210213; J. High Energy Phys. **0101** (2001) 010; G. Marchesini *et al.*, *Comput. Phys. Commun.* **67** (1992) 465.
- [23] S. Frixione and B. Webber, J. High Energy Phys. **0206** (2002) 029; S. Frixione *et al.*, arXiv:1010.0568; B. Webber, talk at CERN, 03/30/2011; S. Frixione, talk at CERN, 05/04/2011.
- [24] D. R. Yennie, S. C. Frautschi, and H. Suura, *Ann. Phys.* **13** (1961) 379; see also K. T. Mahanthappa, *Phys. Rev.* **126** (1962) 329, for a related analysis.
- [25] B.F.L. Ward and S. Jadach, *Mod. Phys. Lett. A* **14** (1999) 491.
- [26] J.G.M. Gatheral, *Phys. Lett. B* **133** (1983) 90.
- [27] A. Mukhopadhyay *et al.*, to appear.
- [28] G. Altarelli, R.K. Ellis and G. Martinelli, *Nucl. Phys. B* **157** (1979) 461, and references therein.
- [29] B. Humpert and W.L. Van Neerven, *Nucl. Phys. B* **184**(1981) 225, and references therein.
- [30] G. 't Hooft, *Nucl. Phys. B* **61** (1973) 455, and references therein.
- [31] S. Alioli *et al.*, J. High Ener. Phys. **1006** (2010) and references therein.